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DIFFUSION OF GASEOUS EMISSIONS IN THE TURBULENT WAKE OF A PIPE

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The concentration of gas effluxing through a circular pipe into an external flow is determined as a function of the distance on the basis of theoretical and experimental data for nitrogen, hydrogen, and helium for small ratios of the efflux and flow velocities, when the jet regime of diffusion degenerates into diffusion in the region of a turbulent wake of the source.

The distribution of the concentration of a gaseous jet diffusion into a cross flow for values of the hydrodynamic parameter $1 \le q \le \infty$ is described in [1]. However, the values of $0 \le q \le 1$, when the gas or a mixture of gases effluxes into an external flow whose velocity is relatively low, are also of interest. In this case the discharged gas diffuses in the region of the turbulent wake of the source (Fig.1). The solution of the equation of turbulent diffusion for a stationary point source with constant turbulent diffusion coefficient and constant velocity of the cross flow has the following form [2]:

$$m(x, y, z) = \frac{M}{4\pi K x} \exp\left[-\frac{u_a (y^2 + z^2)}{4Kx}\right]. \tag{1}$$

This formula gives the following expression for the on-axis concentration, which is maximum in the section x = const:

$$m_{\max}(x, 0, 0) = \frac{M}{4\pi K x}. \tag{2}$$

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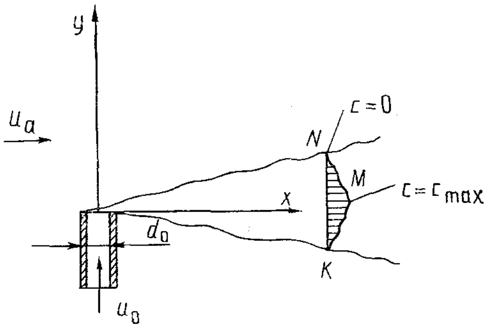

Fig. 1. Diagram of the diffusion of a gaseous discharge: the wavy line indicates the boundary of propagation of the discharge; the curve NMK depicts the concentration in the symmetry plane x–y for a fixed value of x.

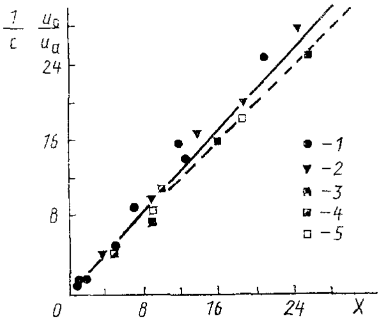

Fig. 2. Distribution of the on-axis concentration in generalized coordinates: 1) hydrogen, d_0 = 20.8 mm, u_0 = 9.1 m/sec, u_a = 9.5 m/sec, q = 0.06; 2) helium, d_0 = 10 mm, u_0 = 18.7 m/sec, u_a = 8.4 m/sec, q = 0.7; 3) nitrogen, d_0 = 10 mm, u_0 = 8.4 m/sec, u_a = 8.4 m/sec, q = 1.0; 4) nitrogen, d_0 = 20 mm, u_0 = 1.9 m/sec, u_a = 3.5 m/sec, q = 0.3; 5) nitrogen, d_0 = 20 mm, u_0 = 4.5 m/sec, u_a = 8.4 m/sec, q = 0.3; the solid straight line was computed from the formula (6) and the dashed straight line was computed from the formula (11) with q = 1.

The mass flow rate through the opening of the pipe is

$$M = \rho_0 u_0 \frac{\pi d_0^2}{4}. \tag{3}$$

In the region of the turbulent wake of the pipe it can be assumed, in accordance with Prandtl's mixing-length hypothesis [2], that the turbulent-diffusion coefficient is given by

$$K = k u_a d_0. \tag{4}$$

Substituting the expressions (4) and (3) into Eq. (2), we have

$$c = \frac{m_{\max}}{\rho_0} = \frac{u_0}{u_a}\frac{1}{16k}\frac{1}{x^*}. \tag{5}$$

The theoretically derived relation (5) is confirmed by the experimental data for hydrogen, nitrogen, and helium emissions with $q \le 1$, presented in Fig. 2 in dimensionless coordinates. The experimental procedure is described in [1]. As one can see from Fig. 2, the empirical dependence has the form

$$\frac{1}{c}\frac{u_0}{u_a} = 1{,}06x^*. \tag{6}$$

Comparing the formulas (5) and (6) we obtain the value

$$k = 0{,}066. \tag{7}$$

In accordance with Eq. (4) the turbulent-diffusion coefficient is

$$K = 0{,}066u_a d_0. \tag{8}$$

The empirical coefficient k in Eq. (8) is approximately two times smaller than the coefficient, presented in [2], in the analagous equation for the turbulent-diffusion coefficient in the turbulent wake behind a cylinder. This can be explained by the fact that in the present work the turbulent-diffusion coefficient is determined for the end of the cylinder (pipe) in the transitional region of the turbulent wake.

For $q = 1$ the semi-empirical formula (6) must match with the empirical relation, obtained in [1], for the distribution of the on-axis concentration in the jet discharge. For isothermal efflux, this relation has the form

$$\left(\frac{1}{c} - 1\right)\sqrt{\frac{\rho_a}{\rho_0}}\,q^{1/6} = x^* - x^*_{\text{in}}. \tag{9}$$

At a sufficienlty large distance from the pipe opening the negative terms on the left- and right-hand sides of the expression (9) can be neglected:

$$\frac{1}{c} = \sqrt{\frac{\rho_0}{\rho_a}}\,q^{-1/6}x^*. \tag{10}$$

Multiplying the left- and right-hand sides of (10) by the ratio u_0/u_a gives

$$\frac{1}{c}\frac{u_0}{u_a} = q^{1/3}x^*. \tag{11}$$

For $q = 1$ formula (11) is practically identical to the relation (6); this provides additional confirmation of the validity of the calculations performed.

It is interesting to note that the expression (6), describing the limiting state of efflux as $q \to 0$, is similar to the expression for the other limiting state $q \to \infty$, obtained for a turbulent jet in a stationary external space [1]:

$$\frac{1}{c}\sqrt{\frac{\rho_a}{\rho_0}} \approx 0{,}225y^*. \tag{12}$$

In both cases the diffusion process does not depend on the hydrodynamic parameter q. This can be explained by the fact that as $q \to 0$ the energy for the turbulent diffusion comes only from the external flow, while in the case $q \to \infty$ the energy is supplied by the effluxing jet.

The results of this work can be used in fields of technology where it is necessary to calculate the diffusion of a single gas, blown through a pipe or a pipe connection into a flow of a different gas.

NOTATION

x and y, horizontal and vertical coordinates; $q=\rho_0 u_0^2/\rho_a u_a^2$, hydrodynamic parameter; ρ_0 and ρ_a, gas density and the air density; u_0 and u_a, gas velocity and the air velocity; $x^* = x/d_0$, $y^* = y/d_0$, dimensionless coordinates; M, mass flow rate through the pipe, kg/sec; m, mass concentration, kg/m^3; c, volume concentration along the x-axis (as $q \to 0$) or the y-axis (for $q \to \infty$); K, turbulent-diffusion coefficient, m^2/sec; k, coefficient of proportionality; and x^*_{in}, dimensionless initial section of the jet.

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NONLINEAR PROBLEM OF HEAT CONDUCTION FOR A HEAT-SENSITIVE SPHERE

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The nonlinear problem of heat conduction is solved for a sphere which exchanges heat through its surface with the external medium according to Newton's law. The temperature field is investigated.

Consider a heat-sensitive sphere which has an initial temperature t_{in} and through whose spherical surface r = R heat is exchanged with the external medium, whose temperature jumps from 0 to t_0. The nonstationary temperature field determined by a prescribed heating action is determined by solving the following boundary-value problem:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\lambda(t)\frac{\partial t}{\partial r}\right] = c_v(t)\,\dot{t}, \tag{1}$$

$$t|_{\tau=0} = t_{\text{in}}, \left.\frac{\partial t}{\partial r}\right|_{r=0} = 0, \left.\left\{\lambda(t)\frac{\partial t}{\partial r} + \alpha\left[t - t_0 S_+(\tau)\right]\right\}\right|_{r=R} = 0. \tag{2}$$

Introducing Kirchhoff's substitution

$$\vartheta = \frac{1}{\lambda_0}\int_{t_{\text{in}}}^{t}\lambda(\zeta)\,d\zeta \tag{3}$$

and making the assumption that $c_V(t)/\lambda(t) = a \cong \text{const}$, we convert the nonlinear boundary-value problem (1)-(2) into the form

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial \vartheta}{\partial r}\right] = \frac{\dot{\vartheta}}{a}, \tag{4}$$

$$\vartheta|_{\tau=0} = 0, \left.\frac{\partial \vartheta}{\partial r}\right|_{r=0} = 0, \left.\left\{\lambda_0\frac{\partial \vartheta}{\partial r} + \alpha\left[t - t_0 S_+(\tau)\right]\right\}\right|_{r=R} = 0, \tag{5}$$

where λ_0 is the reference value of the thermal conductivity.

The boundary condition at the boundary r = R contains the function t(R, τ), i.e., the solution of the desired nonlinear problem, taken on the boundary surface. It is obvious that t(R, τ) is a function of time only. For this reason, we approximate it with the help of splines of zero order, i.e., with the help of asymmetric unit step functions, as follows:

$$t(R, \tau) = t_{\text{in}} + \sum_{i=1}^{m} t_i S_-(\tau - \tau_i), \tag{6}$$

where t_i, where i = 1, ..., m, are, as yet, unknown values, and τ_i are the partition points on the straight line (0, τ).

We note that the representation (6) is adequate for determining the step functions introduced in [1]. Since any continuous and monotonic function is the uniform limit of step functions, the expression (6) is legitimate and correct.

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